A MODEL FOR THE EROSION ONSET OF BEDLOAD TRANSPORT

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Erosion is central to geomorphology, and takes place when fluid shearing forces overcome the gravitational forces acting on sediments. Although many systems, like gravel rivers, lie close to the erosion threshold where these forces balance, the microscopic description of this transition is debated. We introduce a novel model where interacting particles are channelled by disorder, which predicts the spatial erosion pattern to be organized in a fractal manner with novel exponents. It justifies the linear relation between the erosion flux and the excess shear stress near threshold, and leads to analogies with plastic depinning in superconductors. More generally, it indicates that the spatial organization of the erosion flux can be used to distinguish competing theories.

In particular, we study theoretically the erosion threshold of a granular bed forced by a viscous fluid. We first introduce a novel model of interacting particles driven on a rough substrate. It predicts a continuous transition at some threshold forcing θ_c, beyond which the particle current grows linearly $J \sim \theta - \theta_c$. The stationary state is reached after a transient time $t_{\rm conv}$ which diverges near the transition as $t_{\rm conv} \sim |\theta - \theta_c|^{-z}$ with $z \approx 2.5$. Both features agree with experiments. The model also makes quantitative testable predictions for the drainage pattern: the distribution $P(\sigma)$ of local current is found to be extremely broad with $P(\sigma) \sim J/\sigma$, spatial correlations for the current are negligible in the direction transverse to forcing, but long-range parallel to it. We explain some of these features using a scaling argument and a mean-field approximation that builds an analogy with q-models. We discuss the relationship between our erosion model and models for the plastic depinning transition of vortex lattices in dirty superconductors, where our results may also apply [1].

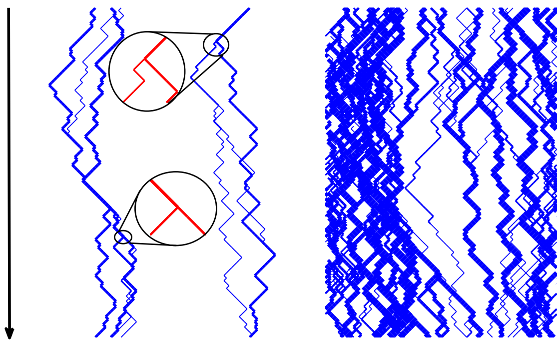

Figure 1. Examples of drainage pattern just below θ_c (Left) and above (Right). The black arrow shows the downhill direction. The thickness of the lines represents $\sigma_{i\to j}$ in logarithmic scale. A few examples showing splitting events are magnified on the left. Here $W = 45$ and $L = 128$, and $J > 0$ even below θ_c due to finite size effects.

References

References

[1] Le Yan, Antoine Barizien, and Matthieu Wyart. A model for the erosion onset of a granular bed sheared by a viscous fluid. *arXiv preprint arXiv:1505.03029*, 2015.