DIRECT IMPLICIT ALGORITHM FOR THE SOLUTION OF ADVECTION-DIFFUSION EQUATION ON A SPHERE

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A numerical algorithm for the solution of advection-diffusion equation on the surface of a sphere is suggested. The velocity field on a sphere is assumed to be known and non-divergent. The discretization of advection-diffusion equation in space is carried out with the help of the finite volume method, and the Gauss theorem is applied to each grid cell. For the discretization in time, the symmetrized double-cycle componentwise splitting method and the Crank-Nicolson scheme are used. The numerical scheme is of second order approximation in space and time, correctly describes the balance of mass of substance in the forced and dissipative discrete system, and is unconditionally stable. In the absence of external forcing and dissipation, the total mass and L_2-norm of solution of discrete system is conserved in time. The one-dimensional periodic problems arising at splitting in the longitudinal direction are solved with Sherman-Morrison's formula and Thomas's algorithm. The one-dimensional problems arising at splitting in the latitudinal direction are solved by the bordering method that requires a prior determination of the solution at the poles. The resulting linear systems have tridiagonal matrices and are solved by Thomas's algorithm. The suggested method is direct (without iterations) and rapid in realization. It can also be applied to linear and nonlinear diffusion problems, and some elliptic problems on a sphere.

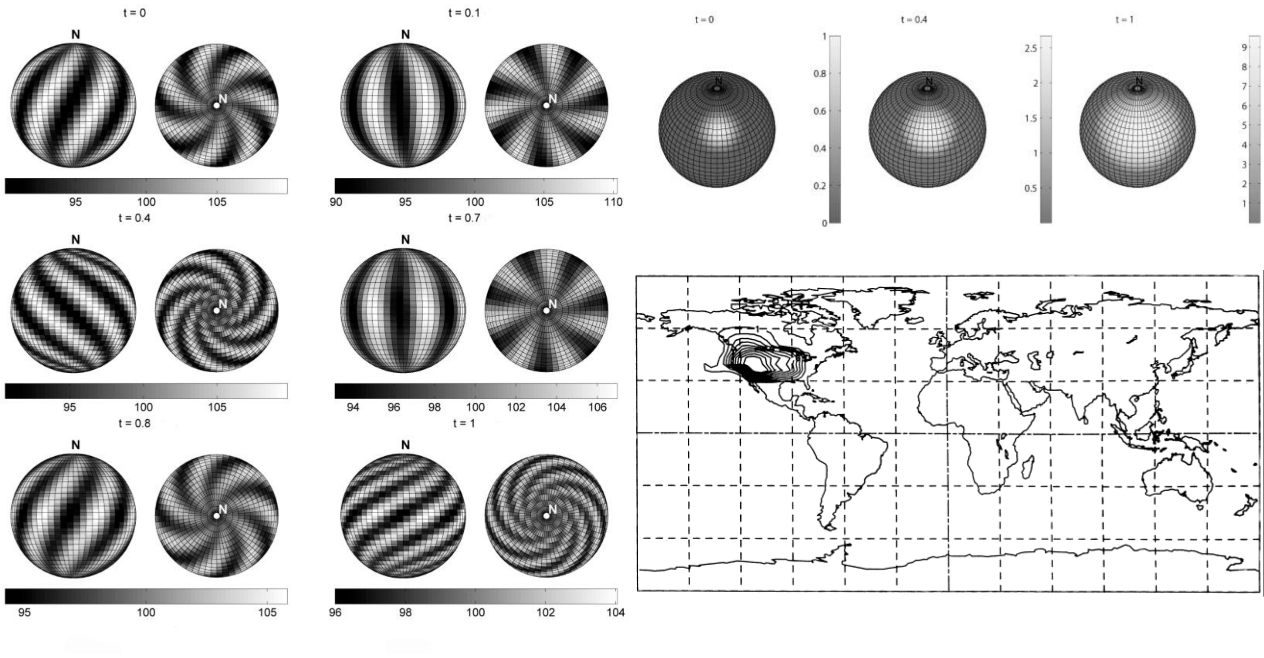

Figure 1. Nonlinear spiral waves, side and top view (a); The HS-regime of nonlinear combustion, t=0, 0.4, 1.0 (b); Solution of adjoint advection-diffusion problem for USA territory, $t = T - \tau / 2$ (T = 60 days, τ = 10 days) (c).

References

[1] Yuri N. Skiba. A non-iterative implicit algorithm for the solution of advection-diffusion equation on a sphere, *Intern. J. Numer. Methods in Fluids*. **78** (5), 257 (2015).