ENVELOPE BROADENING AND SCATTERING ATTENUATION OF A WAVELET IN RANDOM MEDIA HAVING A POWER-LAW SPECTRA

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Seismograms of small earthquakes show scattering effect of random heterogeneities in the lithosphere (e. g. Sato et al. 2012). As shown in Figure 1 (a), we see a delay of the maximum peak from the S-onset and a broadened envelope of S-wavelet larger than the source duration. Peak delay measurements offer fruitful information about lithospheric heterogeneities in relation with seismo-tectonic settings. In Japan arc, strong heterogeneities are found especially beneath Quaternary volcanoes (Takahashi et al. 2007). We also take notice of power-law spectra of velocity well log data, which suggests that random velocity fluctuations have power-law spectra even in seismic spectral range. Here, we study the mathematical aspect of the envelope of a scalar wavelet in von Kármán type random media. Figure 1 (b) shows an example of power spectral density function (PSDF) of random velocity fluctuation. When the center wavenumber of the wavelet is in the power-law spectral range, we propose to divide the random medium spectra into two parts taking the center wavenumber as a reference, and then, we study their scattering contribution to envelope broadening and amplitude attenuation [1]. For the wave propagation through the long-scale (low-wavenumber spectral) component of random media, we may apply the parabolic approximation to the wave equation, which well describes scattering in a narrow angle around the forward direction. Using the Markov approximation, which is a stochastic extension of the phase screen method, we can analytically synthesize the mean square (MS) wavelet envelope G_{M0} for an impulsive source radiation. The resultant envelope duration increases according to the second power of travel distance r; however, the time integral $\int 4\pi r^2 G_{M0} dt$ is conserved irrespective of r. Wide angle scattering caused by the short-scale (high-wavenumber spectral) component of random media contributes to the attenuation of wave amplitude. We use the total scattering coefficient of the short-scale component g_{S0} given by the Born approximation as a measure of scattering attenuation per distance. Multiplying the exponential scattering attenuation factor $\mathrm{Exp}(-g_{S0}V_0t)$ by the MS envelope derived by the Markov approximation G_{M0}, we can synthesize the MS envelope G_{M0S} which reflects the scattering contribution of all the spectral components of random media. Figures 1 (c) and (d) show synthesized MS wavelet envelopes with geometrical correction for different center frequencies at a 100 km distance by way of example, where solid and dashed lines are with and without scattering attenuation correction. When the PSDF has a small role-off as $\kappa = 0.1$, the envelope broadening is large and increases with frequency, and the scattering attenuation is strong and increases with frequency. When the PSDF has a steep role-off as $\kappa = 1$, however, the envelope broadening is small and frequency independent, and scattering attenuation is weak. The proposed synthesis is fully analytic; therefore, it can be a theoretical basis for the evaluation of random heterogeneity of the earth medium from the envelope broadening of seismograms in short periods. In the above envelope synthesis, wide angle scattering is considered only for the amplitude attenuation effect. For explaining coda envelopes, it will be necessary to incorporate multiple wide-angle scattering process in the framework of the radiative transfer theory.

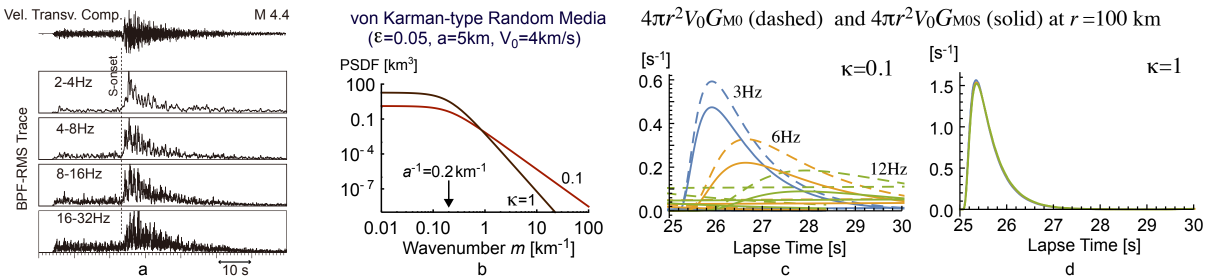

Figure 1. (a) RMS seismogram envelopes (Saito et al. 2002). (b) Example of PSDF of random velocity fluctuation. (c) and (d) synthesized MS envelopes with geometrical correction for an impulsive source radiation [1].

References

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