Faults are rough at all scales and can be described as self-affine fractals. This deviation from planarity results in geometric asperities and a locally heterogeneous stress field, which affect the nucleation and propagation of shear rupture. We study this effect numerically at the scale of small earthquakes, in which realistic geometry and friction law parameters can be incorporated in the model. We aim to understand the effect of roughness on faults with $L \sim 10 – 200$ m. At this scale we can choose the minimum roughness wavelength, $\lambda_{\text{min}}$, to be the size of lab samples ($5 – 10$ cm) and thus use the observed lab-scale slip or rate based friction laws without modifying the constitutive parameters, assuming that the experimental data already include the effects of smaller wavelengths of roughness. Moreover, using a variable time step size, we gradually increase the remote stress and let the rupture nucleate spontaneously, rather than introducing artificial procedures to nucleate a seismic event.

Numerically, maintaining $\lambda_{\text{min}}$ and consequently the smallest element size, $\Delta x$, fixed while increasing the fault length poses two challenges. First, keeping $\Delta x$ fixed is computationally expensive. Second, the slip increases with increasing fault length and the assumption of small slip relative to the size of the elements is not valid. To overcome the first challenge, we refine the mesh near the fault, using hanging nodes. To overcome the second challenge, we use the Mortar finite element method [Bernardi et al., 1994; Wohlmuth, 2000], in which non-matching meshes are allowed across the fault and the contacts are continuously updated. We introduce slip weakening and rate and state friction laws into the method and study both the nucleation and propagation of shear rupture, using variable time steps with a quasi-static scheme for the inter-seismic stage and a dynamic implicit Newmark scheme for the co-seismic stage.

For a static benchmark, we demonstrate that the method predicts accurately the stresses and displacements along a fault with a non-matching grid due to a uniform stress drop. We also design a benchmark problem to show that the method accurately models the behavior of the friction coefficient in response to a change in the slip rate on a fault governed by a rate and state friction law.

Simulations of a 10 meter long horizontal fault with different amplitude roughness and a slip-weakening friction law show the significant effect of roughness on: (1) Slip on the fault and consequently the seismic moment; (2) Stress drop; (3) Rupture properties, such as rupture velocity, breakdown zone, and the observed relation between shear stress and slip; and (4) Different stages in the nucleation processes. For example, with the adopted spontaneous nucleation procedure, the experiment-based nucleation model of Ohnaka [2000] is observed also in the simulations (Fig. 1), and important quantities regarding the nucleation and propagation of the rupture can be measured.

**Key words** Fault roughness, Mortar finite elements, shear rupture.

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**Figure 1.** (a) A rupture nucleation model (after Ohnaka [2000]). (b) Observed nucleation and propagation of a shear rupture on a rough fault. The markers are shown only for nodes where the friction coefficient at a given time is $\mu_d < \mu < \mu_c$.

**References**

