A HYDROSTATIC MULTILAYER MODEL WITH THE $\mu(I)$-RHEOLOGY FOR DRY GRANULAR FLOWS

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Granular flows have been widely studied in recent years because of their importance in industrial processes and geophysical flows such as avalanches, debris or rock avalanches, landslides, etc. The behaviour of real geophysical flows is very complex due to topography effects, heterogeneity of the material involved, etc. Hence it becomes a major challenge to develop a mathematical model able to reproduce correctly the dynamic of such flows. One of the issues recently analyzed by some authors (see [7, 3]) on granular collapse is the different behaviour of the runout distance in the presence or not of an erodible bed. They showed experimentally a significant increase of the runout distance and flow duration with increasing thickness of the erodible bed. The question remains as to whether this behaviour can be reproduced by granular flow models.

Due to the high computational cost of solving the full 3D Navier-Stokes equations, granular flows have often been simulated using depth-averaged shallow models. But, as it is well known, with this kind of approximation one lose the vertical nature of these flows. Indeed, the introduction of the multilayer models is aimed at resolving this lack inherent to the simple one-layer shallow models, because they take into account the change of the velocity field in the normal direction to the topography.

On the other hand, the rheology of granular flows has been usually described by viscoplastic laws and namely by the so-called $\mu(I)$ rheology [1, 4, 6]. The $\mu(I)$-rheology, introduced by Jop et al. [5], considers a Drucker-Prager plasticity criterion and the shear stress tensor is assumed proportional to the normal one, where the variable coefficient $\mu(I)$ depends on the inertial number I.

Following [2], in this work we present a multilayer approach of the $\mu(I)$-rheology model with hydrostatic pressure. The multilayer approach allows us to recover the vertical profile of the velocity for these flows, and to improve the approximation of the vertical diffusion term in the model.

Several comparisons of the numerical results with different Bagnold flows and with laboratory data will be presented. By comparing with the laboratory data for a granular collapse experiment (see [7]) we show that our model well reproduces the influence of the erodible bed on the flow dynamics and deposit.

References

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