BOUNDARY CONDITION OPTIMAL CONTROL PROBLEM IN LAVA FLOW MODELLING

A. Ismail-Zadeh[1 2 3], A. Korotkii[1 4 5], D. Kovtunov[1], O. Melnik[1 6] & I. Tsepelev[1 5]

[1] *Karlsruhe Institute of Technology, Institute of Applied Geosciences, Karlsruhe, Germany*
[2] *Russian Academy of Sciences, Inst. of Earthquake Prediction Theory and Math. Geophysics, Moscow, Russia*
[3] *Institut de Physique du Globe, Paris, France*
[4] *Institute of Mathematics and Computer Sciences, Ural Federal University, Yekaterinburg, Russia*
[5] *Institute of Mathematics and Mechanics, Russian Academy of Sciences, Yekaterinburg, Russia*
[6] *Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russia*

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Rapid development of ground based thermal cameras, drones and satellite data allows getting repeated thermal images of the surface of the lava flow. Available instrumentation allows getting a large amount of data during a single lava flow eruption. These data require development of appropriate quantitative techniques to link subsurface dynamics with observations. We present a new approach to assimilation of thermal measurements at lava's surface to the bottom of the lava flow to determine lava's thermal and dynamic characteristics.

We study a problem of steady-state fluid flow with known thermal conditions (e.g., measured temperature and the heat flux at the surface of lava flow) at upper segment of the model boundary and unknown conditions at its lower segment. This problem belongs to a class of boundary condition optimal control problems and can be solved by assimilation of the data from the upper to lower boundary using direct and adjoint models. We derive analytically the adjoint model and test the cost function and its gradient, which minimize the misfit between the known thermal condition and its model counterpart. Using optimization algorithms, we iterate between the direct and adjoint problems and determine the missing boundary condition as well as thermal and dynamic characteristics of the fluid flow. The efficiency of optimization algorithms – Polak-Ribiere conjugate gradient and the limited-memory Broyden-Fletcher-Goldfarb-Shanno algorithms – have been tested with the aim to get a rapid convergence to the solution of this inverse ill-posed problem.

Numerical results show that in the case of smooth input data lava temperature and velocity can be determined with a high accuracy. A noise imposed on the smooth input data results in a less accurate solution, but still acceptable below some noise level. The proposed approach to assimilate measured data brings an opportunity to estimate thermal budget of the lava flow.

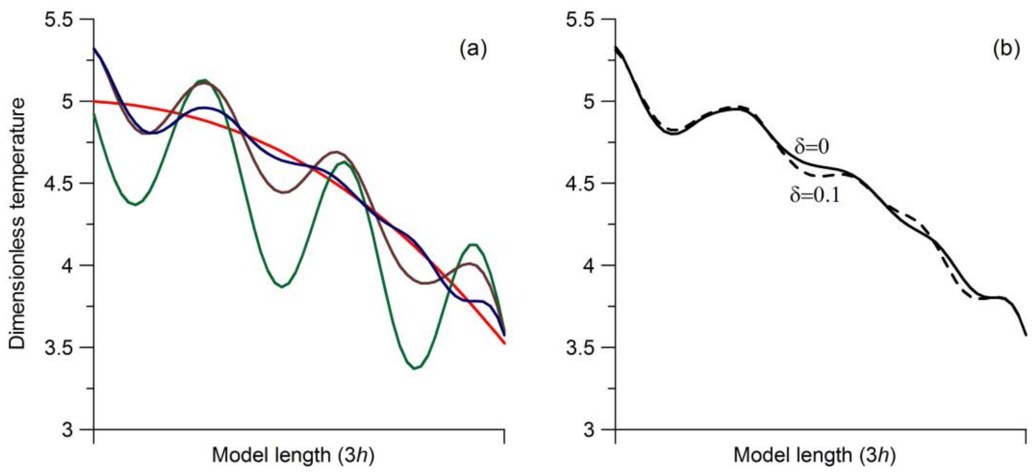

Figure 1. Reconstruction of the temperature at the lower segment of the model boundary (*a*). The red curve corresponds to the target temperature, the green curve to the guess temperature, the brown curve to the temperature after 5 iterations, and the blue curve to the temperature after the 10 iterations. The reconstructed temperature after 10 iterations (*b*) in the case of no noise in the heat flow at the upper segment of the model boundary (solid line; the blue curve in *a*) and in the case of noise in the heat flow (dashed line).