A PHASE DIAGRAM FOR FLUID-SHEARED GRANULAR BEDS

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Fluid flowing laterally over a granular bed exerts a shear stress on the grains, and a sufficiently fast flow can transport grains with the fluid, a process that shapes much of the natural world. The nature of the boundary between systems with and without grain motion has been studied for many decades but is still not clearly understood, as it involves nontrivial coupling between several complex physical processes: turbulent fluid flow above the bed, Darcy flow inside the bed, and the yield stress of granular materials of varying shape and size. This large, complex parameter space represents a significant challenge for the field, both to connect the results from the literature (experimental, computational, or field data) into a unified description, as well as to identify the primary physical processes that are responsible for controlling various aspects of sediment transport (and related problems).

Surprisingly, the onset of grain motion in these systems is fairly well captured by two dimensionless numbers. The Shields number Θ is the ratio of the shear stress exerted by the fluid at the bed surface to the downward gravitational stress acting on these grains. The particle Reynolds number Re_p gives the ratio between grain inertia and viscous damping from the fluid. Several recent reviews have shown collections of data, as shown in Fig. 1(a), from experiments and field studies, where the critical Shields number $\Theta = \Theta_c$ required for grains to move is plotted versus Re_p; this data clusters around a curve with plateaus Θ_c^l and Θ_c^h at low and high Re_p, where $\Theta_c^l > \Theta_c^h$, and there is a transitional region between these two limits near $\mathrm{Re}_p \approx 1$. At present, the dominant mechanisms in shaping this curve and the relative importance of each—e.g., friction, turbulence, grain shape, grain size—are not well understood.

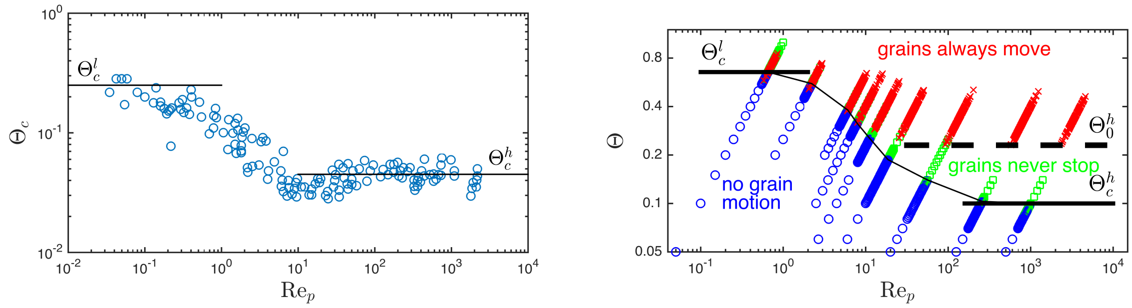

Figure 1. (left panel) A collection of experimental and field data from [1] showing how the critical Shields number Θ_c varies with Re_p; plateaus Θ_c^l and Θ_c^h are marked, representing behavior at low and high Re_p. (right panel) Typical result from simplified two-dimensional DEM simulations [2] with a model fluid that exerts a shear force on a granular bed. Blue circles and green squares represent mobile systems that did and did not stop, respectively. Red crosses show the value of Θ where a static system becomes mobilized. The solid line connecting Θ_c^l and Θ_c^h is $\Theta_c(\mathrm{Re}_p)$, which marks the minimum Θ that can maintain grain motion indefinitely at varying Re_p. The dashed line shows Θ_0^h, the minimum value of Θ at which static systems can become mobilized at high Re_p. As system size grows, all beds at high Re_p will mobilize at $\Theta = \Theta_0$.

Here, I will describe a continuing line of research, where we seek to identify the basic physical mechanisms that govern sediment transport (and related problems) by performing numerical simulations (DEM) of a system with only the most crucial elements. We study granular beds with a free boundary at the surface, as opposed to constant pressure or volume, and the surface grains are thus most susceptible to motion. Instead of constant force or velocity, we apply forcing through a variable fluid flow profile that depends on the state of the grains (relatively large above the bed with a smooth transition to a very small fluid velocity within the bed). The force felt by a grain depends on the difference between the local fluid velocity and the grain velocity, and the strength of the coupling between the fluid flow profile and the grain velocity gives Re_p. The force felt by a static grain at the bed surface gives Θ. We can vary the details of the fluid profile and the grain-grain interactions, but we find a surprisingly robust form for the phase boundaries in our system, as shown in Fig. 1(b), which shows qualitative agreement with the field and experimental data in Fig. 1(a). These boundaries are insensitive to the details of the fluid flow we apply and the grain-scale properties such as shape, friction, and dissipation from intergrain collisions. We find critical-like behavior near these boundaries (e.g., diverging transient time scales), and we find that the transition from static-to-mobile beds is governed by Weibullian weakest link statistics, where global failure is caused by the failure of the weakest region. These results suggest that the details of the granular structure, a factor that has often been neglected in previous descriptions, plays an important role in determining the onset of grain motion in fluid-driven beds.

References

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