SHORELINE DYNAMICS: EXPLICIT FORMULATIONS FROM THE NON-LINEAR PELNARD-CONSIDÈRE EQUATION

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Key words Pelnard-Considère, non-linear diffusion, anti-diffusion, shoreline features, erosion, cuspate, spits.

We tentatively contribute to a better understanding of shoreline dynamics starting from a non-linear Pelnard-Considère equation [3]. In the following, we define the shoreline as being a curve moving in plan view, averaging the location of the interface between land and sea over time scales of the order of days or weeks at least.

Many works analyze how natural or human-controlled shoreline features – such as sand spits, cuspates, flying spits, tombolos – may develop through time [6]. In the past, research activities on this topic strictly developed from a diffusion equation [8], with severe restrictions on the physics embedded. More recent works introduced non-linear effects and successfully modeled the growth of single instabilities or repetitive patterns [1, 4, 2, 5, 7]. Non-linearity is crucial to tackle problems such as 1) the cyclic distribution of geomorphic features alongshore, 2) the spontaneous nucleation of single features, 3) the migration of morphologies, 4) the interactions between shoreline instabilities. Amongst these works, Ashton & Murray [2] suggested that the following equation (S being the plan viewed shoreline position):

$$\frac{\partial S}{\partial t} = \kappa \frac{\partial^2 S}{\partial x^2} \tag{1}$$

where some kind of diffusivity κ depends upon wave breaking angle, was a good expression to model shoreline instabilities. More specifically, they develop a numerical tool solving this equation for the growth of sand spits. From the analysis of the numerical results, they suggest the existence of relationships between the dimensions of geomorphic features and their age.

In this work, we write the following local diffusive/ anti-diffusive shoreline equation:

$$\frac{\partial S}{\partial t} = G_0 \cos 2\delta_0 \frac{\partial^2 S}{\partial x^2} + 2G_0 \sin 2\delta_0 \frac{\partial S}{\partial x}\frac{\partial^2 S}{\partial x^2} \tag{2}$$

where G_0 a longshore diffusivity, δ_0 the deep-water incident wave angle. Obviously, the linearization of this equation leads to the so-called linear diffusion equation of Pelnard-Considère [8].

First, we show that Eqs 1 and 2 are strongly connected although they show important discrepancies, especially the fact that Eq. 2 embeds non-linearity but does not require any information relative to the wave angle at wave breaking, where non-linear effects may develop. We also deeply explore the effects of anti-diffusive effects in the Eq. 2. Then, although Eq. 2 does not offer any trivial general solution, we successfully derive peculiar explicit solutions for the growth of cuspate spits, and we give a formal proof of the existence of an alternative relationship between the dimensions of a cuspate, its age and the surrounding longshore diffusivity; we also provide a solution for the growth of tombolos.

Finally, we present the on-going developments to solve numerically the non-linear Pelnard-Considère equation.

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