RAPID, REPEATABLE PROBABILISTIC INVERSION: THE 'PRIOR SAMPLING' FRAMEWORK

A.P. Valentine, J. Trampert, S.E. Atkins, P.J. Käufl, & R.W.L. de Wit
Universiteit Utrecht, Utrecht, The Netherlands

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Deterministic, linearised imaging techniques—such as those based on least-squares inversion—have contributed a huge amount to our understanding of the Earth. Nevertheless, they have their drawbacks: in particular, developing a proper understanding of model resolution and how uncertainties and systematic errors (such as approximations) propagate into solutions is challenging [1, 2, 3]. This creates a barrier to robust interpretation of results. Monte Carlo methods (based around 'sampling' the relationship between models and data) offer a route to circumventing many of these problems [4] and their use is increasingly widespread.

Typically, such methods rely on 'posterior sampling', where the sampling procedure is optimised towards explaining a given dataset. In many situations, this is sensible—but it brings two potential drawbacks. First, it imposes a fundamental restriction on the rate at which a solution can be obtained: no calculations can begin until observations have been collected. Second, samples cannot easily be 'recycled' in conjunction with a different dataset (perhaps obtained from a second location or point in time). These factors make it difficult to apply posterior sampling in real-time monitoring settings (such as for earthquake early warning), or to analyse local phenomena systematically around the globe (as in, say, receiver function inversion).

We therefore present a new approach, which we describe as 'prior sampling' [5]. Samples are distributed throughout the model space, according to any prior information that might be available. For each sample, the (potentially computationally-expensive) forward model is evaluated to obtain corresponding synthetic data. This data can be corrupted to mimic the effects of observational noise, and to account for any uncertainties due to any approximations or deficiencies in the forward code. The collection of (model, data) pairs obtained in this manner characterise a probability density function (pdf) in the joint data-model space. We assume that this pdf is smooth and continuous, and we fit a parametric representation to the samples. To do this, we employ a particular class of neural network called a 'mixture density network' (MDN) [6]. No observations of real data have been used in setting up this machinery. Once the MDN has been constructed, we are in a position to solve inverse problems repeatedly and rapidly: given a set of observations, the MDN outputs distributions on model parameters within milliseconds, and can be run on a standard desktop PC.

We demonstrate this framework in a range of applications, including earthquake early warning [7], analysis of Earth's radial structure [8], and as a tool for exploratory data mining, to investigate the parameters governing mantle convection [9]. Results are consistent with those obtained via posterior methods, although the less-targeted nature of sampling tends to result in more conservative uncertainty estimates. Nevertheless, we believe the approach is well-suited to inversion in circumstances where posterior approaches are infeasible.

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