THE FRICTIONAL FREQUENCY RESPONSE AND MODEL IDENTIFICATION

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The fields of earthquake mechanics and vibration engineering both provide problems in dynamics in which interfacial friction plays a key role. To make progress with modelling the phenomena in either field a reliable constitutive friction law is required in order to allow an accurate prediction of self-excited instability threshold and vibration levels. Although different friction laws have been proposed over the years, there is no consensus yet when it comes to problems involving relatively high frequency vibration; a frequency domain which may have been overlooked in experimental tribology and seismology. Recent advances [2, 3, 4] show that the concept of the frictional frequency response reveals new dynamic features of the frictional interface for a perturbed steady-sliding contact and is the required input for predicting friction-induced vibration by using linearised stability analysis.

The frequency response function $\beta(\omega) := F^\prime/\omega^\prime$ is constructed by measuring the perturbation in the frictional force $F^\prime$ divided by the perturbation of the sliding speed $\omega^\prime$ as a function of frequency. Such a measurement can be achieved by a pin-on-disc device (Fig. 1a) connected to a piezo actuator driven with band-limited random noise and used to superimpose a small fluctuation in sliding speed to the contact point. Both force and velocity fluctuation are measured in the frictional and normal directions by a package of sensors installed close to the contact region. For different material combinations, it has been found [2, 4] that the measured frequency response is a frequency dependent complex number (Fig. 1d), which indicates that the friction force must also depend on variables with a dynamic evolution law. Up to mid-high frequencies, such measurements proved to be more repeatable than measurements of the steady-state friction curve (Fig. 1b-c). Previous efforts to explain such measurements from a theoretical model have failed. However, an enhanced rate-and-state model, that takes elastic deformations near the interface into account through a contact stiffness $k_\phi$ [1, 4], is shown to match the measurements remarkably well over a parameter space that covers a range of normal forces, of sliding speeds and frequencies (Fig. 1b-d).

A systematic optimization methodology allows discriminating among possible variants of the model, and then identifying the model parameter values (Fig. 1d). The results suggest that the validity range of the enhanced rate-and-state model can be extended up to mid-high frequencies. It is worth underlining that the measurements done so far are mainly proofs of concept of this innovative experimental technique and the subsequent model-fitting process. Such a methodology could be extended to the characterisation of any frictional interface and the information gained could then be used for a linearised stability analysis for friction-induced vibration and stick-slip instability prediction. Future investigation could involve developing the proposed methodology from localised frictional contact to the case of extended contact, which would also relate to tests frequently performed in experimental seismology, by the use of an array of strain gauges and accelerometers placed near a sliding interface.

![Figure 1](image_url)

Figure 1. a) Pin-on-disc rig; b) Velocity-strengthening steady-state friction coefficient; c) Frictional frequency response [$|\beta(\omega)|$] for different disc speed $v_0$ (Nylon pin - glass disc at $N = 20$ N): $v_0$ is increased stepwisenly then decreased again showing a striking repeatability of the measurement; d) Corresponding Nyquist plot: the red lines indicate the model prediction after fitting the whole data matrix (for various disc speeds and normal forces), the dashed lines indicate single curve fittings. In this experiment, the memory length $\phi$ appears to be linearly dependent on $v_0$ regardless of the model, suggesting that a rather constant relaxation time-scale $\tau_\phi$ for the interficial state $\phi$ is more appropriate.

Identified rate-and-state parameters: $a = 0.0196$, $b = 0.0094$, $t_\phi = 0.00075$ s and $k_\phi = 0.27N^{0.349}$ in MN m.

References