# NON-LINEAR RELATIONSHIP BETWEEN VISCOUS DISSIPATION AND CONVECTIVE HEAT FLUX

T. Alboussière, J. Curbelo, L. Duarte, S. Labrosse, F. Dubuffet & Y. Ricard
*Laboratoire de Géologie de Lyon, France*

*Key words* Rayleigh-Bénard convection; Compressible convection; Equation of state.

The Boussinesq model of natural convection is a successful approximation that has been used extensively. This is a model containing two sources of non-linearities: heat convective transport and inertia. It follows that the solutions exhibit instabilities and chaotic behaviour at high Rayleigh numbers. However, a simple (time-averaged) linear relationship persists between the total viscous dissipation and the convective heat flux. This is due to the structure of the Boussinesq approximation [1] but becomes thermodynamically inconsistent when compressibility effects are significant [2]. This is the case for convection in the interior of planets where the effect of pressure differences on density is typically larger than the effect of temperature differences. This applies also to the atmosphere of the Earth, to giant gas planets and stars. When the Boussinesq model is abandoned, one does not replace it with the fundamental equations from mechanics and thermodynamics, because this model allows acoustic waves to develop. They can be very fast compared to the typical timescale of the convection and would make numerical solutions impossible to obtain. So-called anelastic models have been developed where acoustic waves are absent and which are supposed to be a fair representation of general convective phenomena. In our group, we have compared numerical solutions of the full set of equations to those of an anelastic model. We made our life easier by considering a "fluid" of infinite Prandtl number, an assumption well justified for the dynamics of the mantle of the Earth. With an infinite Prandtl number, acoustic waves do not develop and numerical calculations are not too time-consuming. In particular, we have looked at the relationship between dissipation and convective heat flux (see Fig. 1). There is indeed a departure from the linear Boussinesq relationship but that departure is only partially recovered by the anelastic model. I will discuss the possible reasons for this partial failure of the anelastic model and its implications.

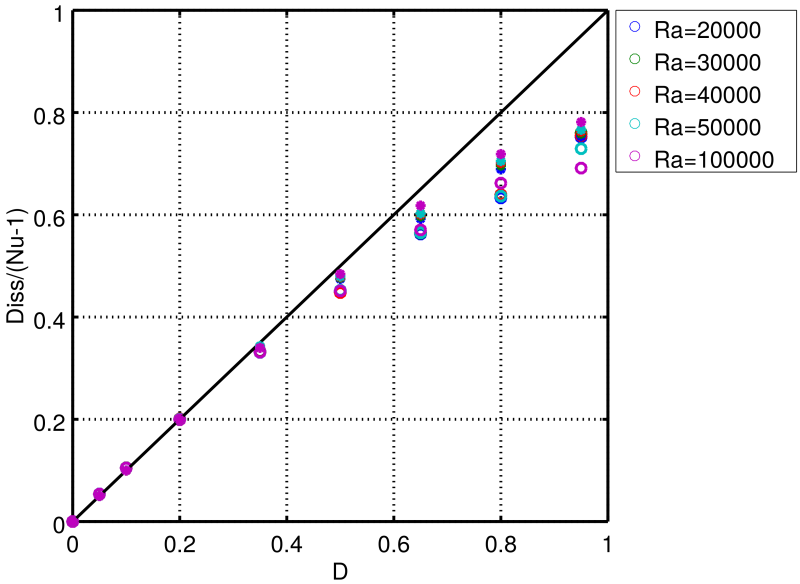


**Figure 1.** Ratio of viscous dissipation to convective heat flux, versus the dimensionless dissipation number $D = \alpha g L / c_p$. Full symbols correspond to the anelastic model while empty symbols correspond to the complete compressible equations.

## References

[1] C.R. Doering and J.D. Gibbon, *Applied Analysis of the Navier-Stokes Equations*, Cambridge University Press, 1995
[2] T. Alboussière and Y. Ricard, *Reflections on dissipation associated with thermal convection*, Journal of Fluid Mechanics, **725**, 2013