DYNAMICS OF FRICTIONAL SLIP LOCALISATION

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An explanation is proposed of the origins of travelling waves of stick and slip that have been observed between frictional sliding surfaces [4, 1, 3]. Such waves, at very different lengthscales, are thought to underlie both earthquakes' source and the performance of mechanical brakes and dampers. An idealised situation is studied of a long thin elastic plate, subject to uniform shear stress $\bar{\mu}$ and frictional interaction with a rigid flat surface. Seeking deformation that is uniform in the transverse direction, a nonlinear wave equation is constructed under a long-wave assumption. Appearing from global homoclinic or heteroclinic bifurcations and under non-monotonic rate-and-state friction [9, 7, 8] a rich variety of solution types is discovered at intermediate shear stress between pure uniform stick and uniform slip, including: periodic stick-slip wave-trains, isolated pulses of stick or slip and detachment or attachment fronts. Careful consideration of the choice of the friction model and interfacial state kinetics is necessary to capture the full richness of wave types. See Fig. 1. This plethora of behaviours strongly depends on the analytical details of the friction model and may shed new light on the dynamics of earthquake ruptures, in particular with respect to the recent field evidence that suggests seismic slip localizes along a fault patch that is partially locked during the interseismic period [10]. Might a locked fault patch be a slow stick pulse? Finally we emphasise that this study shows that these slip patterns can exist over a wide range of speeds of travelling wave. This may explain the large variability of earthquake duration and frequency spectrum [6, 5]. Future work will study the stability of these localized slip structures and their physical selection.

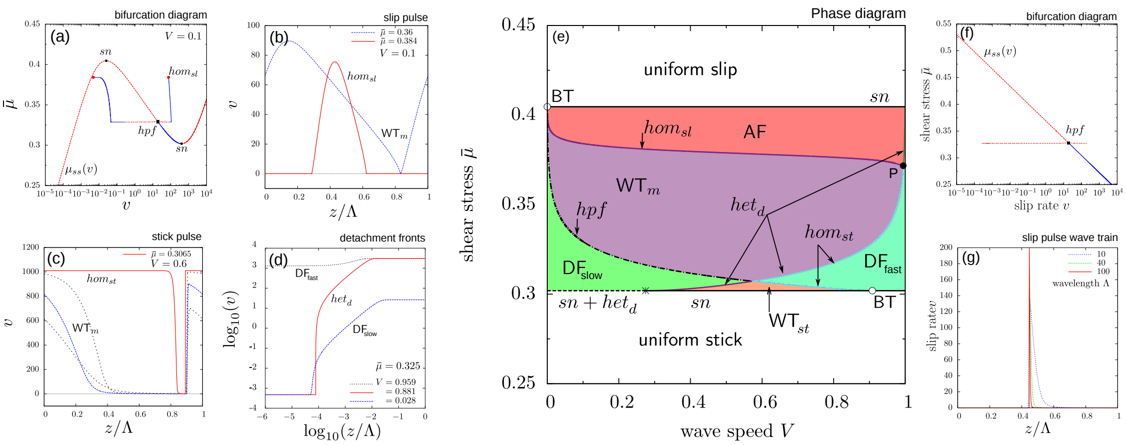

Figure 1: The diversity of travelling localised frictional slip patterns computed from a non-monotonic friction model [8]. (a) Bifurcation diagram showing the growth of wave-trains from a Hopf bifurcation (■), promoted by velocity-weakening friction, leading to the homoclinic orbit (•) of slip pulse (b) reminiscent of the pulses described by Heaton [4]. Such slip pulses are anchored at the equilibrium saddle point lying on the low-velocity-strengthening branch of the steady-state friction curve. (c) The existence of a high velocity strengthening branch of the friction curve $\mu_{ss}(v)$ also allows the existence of 'stick pulse' which corresponds to a narrow travelling 'stick' zone. (d) Heteroclinic orbits connecting the saddle equilibria lying on the low and high velocity-strengthening branches of μ_{ss} correspond to travelling 'detachment' (similar to [3, 2]) or 'attachment' fronts promoting the slab acceleration or deceleration. The location of attachment fronts, which is difficult to compute, is expected to follow very closely the slip pulse locus (hom_{sl}). (e) Phase diagram of travelling wave patterns: homoclinic (pulses) and heteroclinic (fronts) connections exist on lines within the $(V, \bar{\mu})$ parameter space that delineate domains of generic travelling fronts and wave-trains of different types. The precise topology of this bifurcation structure strongly depends on the mathematical details of the friction model, in particular the state evolution equation. Nomenclature: loci of slip pulses (hom_{sl}), stick pulses (hom_{st}), detachment fronts (het_d), Hopf bifurcation locus (hpf); saddle-node bifurcation loci at the local extrema of friction (sn); Stick-slip wave-trains (WT). Generic detachment (DF) and attachment (AF) fronts. Takens-Bogdanov bifurcation point (BT). By contrast the classic monotonic Dieterich-Ruina laws [9] only predict wave trains of slip-pulse solutions in a window of the applied shear stress $\bar{\mu}$ that is exponentially narrow, see (f-g). Note that the travelling waves are moving to the left in these examples. Similar solutions are expected for waves travelling to the right.

References

[1] Behrendt J. et al., J. Sound Vib., **330**:636–651, (2011).
[2] Bouchbinder E. et al., Phys. Rev. Lett., **107**:235501, (2011).
[3] Ben-David O. et al., Science, **330**:211–214 (2010).
[4] Heaton T.H., Phys. Earth Planet Int., **64**:1–20, (1990).
[5] Ide S., Proc Jpn Acad Ser B Phys Biol Sci. **90**:259–277, (2014).
[6] Peng Z. & Gomberg J., Nature Geosci., **3**:599–607, (2010).
[7] Putelat T. et al., J. Mech. Phys. Solids, **58**:27–53, (2010).
[8] Putelat T. & Dawes J., J. Mech. Phys. Solids, **78**:70–93, (2015).
[9] Ruina A.L., J. Geophys. Res., **88**:10,359–10,370, (1983).
[10] Thomas M.Y. et al., J. Geophys. Res., **119**: 5114–5139, (2014).