NUMERICAL SIMULATION OF THE DYNAMICS OF SEDIMENTARY RIVER BEDS WITH A STOCHASTIC EXNER EQUATION,

<u>E. Audusse</u>¹, S. Boyaval², N. Goutal^{2,3}, M. Jodeau³ & P. Ung^{3,4}
¹LAGA, Université Paris 13, Villetaneuse, France
²Lab. Saint-Venant, Chatou, France
³EDF-LNHE, Chatou, France
⁴MAPMO, Univ. Orléans, Orléans, France

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At the scale of a river reach, the dynamics of the river bed is typically modelled in industrial softwares by Exner equation (conservation of the solid mass) with an empirical solid flux of transported sediments, which is a simple deterministic algebraic formula function of i) the sediment physical characteristics (size and mass) and of ii) the averaged hydrodynamical description of the ambient water flow computed through the solution of a Saint-Venant type model. This model has proved useful, in particular through numerical simulations, for hydraulic engineering purposes (like estimating the mass of sediments that is drained through an open dam). Though, the model is also coarse. And its applicability at various space and time scales remains a question of considerable interest for engineers and researcher in sediment transport. In particular, physical experiments from the grain scale to the laboratory scale reveal important fluctuations of the solid flux in given hydrodynamical conditions [1, 2]. This work is a preliminary study of the coupling of a stochastic Exner equation with a Saint-Venant type hydrodynamical model for large scales. Stochastic models with a probabilistic solid flux are currently being investigated by various team, but most often from the viewpoint of theoretical physics at the grain scale [3, 4]. It seems to us that there is still a need for computer experiments in order to investigate how the stochastic approach can be used by engineers that are involved in applied hydraulics studies. In this talk we propose first to study a stochastic Saint-Venant model with perturbed bottom using numerical simulations in an appropriate test case. We exhibit a necessary relation between the bottom fluctuations and the friction coefficient to ensure equilibrium, see Fig. 1 on the left. We also show the convergence toward a steady state and analyze in details the sensibility of this steady state to the input perturbations, with particular attention to hydrodynamic variance and covariance results, see Fig. 1 on the right. In a second step, we introduce a stochastic Saint-Venant-Exner model with a perturbed sediment flux and present some results in this more complex framework. A first part of this work was published in [5].



Figure 1. On the left : Evolution of the friction coefficient as a function of the parameters describing the perturbed bottom - On the right : Correlation of the water height as a function of the smoothness parameter of the perturbed bottom.

References

- E. Lajeunesse, L. Malverti, and F. Charru. Bed load transport in turbulent flow at the grain scale: Experiments and modeling, *Journal of Geophysical Research*, Vol. 115, 2010.
- [2] Recking, A and Frey P. and Paquier, A. and Belleudy, P. and Champagne, J.Y. Bed-Load Transport Flume Experiments on Steep Slopes, *Journal of hydraulic Engineering*, Vol. 134-9, 2008
- [3] C. Ancey, Stochastic modeling in sediment dynamics: Exner equation for planar bed incipient bed load transport conditions, *Journal of Geophysical Research*, Vol. 115, 2010.
- [4] D.J. Furbish, M.W. Schmeeckle, Probabilistic Formulation of the Exner Sediment Continuity Equation and the Bedload Flux for the Case of Uniform Sediment
- [5] E. Audusse, S. Boyaval, N. Goutal and M. Jodeau, P. Ung, Numerical simulation of the dynamics of sedimentary river beds with a stochastic Exner equation, *Esaim: Proceedings and Surveys*, Vol. 48, 2015.