THE SOURCE OF NUMERICAL ERRORS IN SYMPLECTIC INTEGRATION AND PERSPECTIVES IN MULTISYMPLECTIC INTEGRATION FOR THE ELASTIC WAVE EQUATION

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Key words Symplectic integrators, Numerical error, Multisymplectic integrators, Elastic wave equation.

The idea of using geometric integration for long time wave propagation computations in seismology has been proposed for several authors, but no consistent (intrinsically defined) theory has been developed for elastic dynamics, in particular for the elastic wave equation. Some extensions of the multisymplectic formalism from Bridges and Reich [1] and Marsden, Patrick and Skoller [2] have been proposed by some authors, in particular by Jing-Bo [4, 3], and his collaborators. Unfortunatelly, almost all the numerical algorithms in the literature are based on the discretization proposed in [1] and/or [2].
Instead of using the Moser-Veselov discretization for variational integrators [2] or the method of multiple-symplectic forms [1], we started a systematic study of symplectic integration in order to base the construction of intrinsic symplectic algorithms only with the Hamiltonian formalism and symplectic geometry.
In this talk we present a new method for constructing symplectic integrators [5], which enables us to manipulate the numerical Hamiltonian error around a fixed energy level (see Figure 1) [6]. We present the mechanism which produces this error and some conditions for constructing exact symplectic algorithms. We will present our advances in the translation of the method of Liouvillian forms [5] to the multisymplectic formalism and the role the elastic tensor will play for obtaining very accurate methods for solving the elastic wave equation.

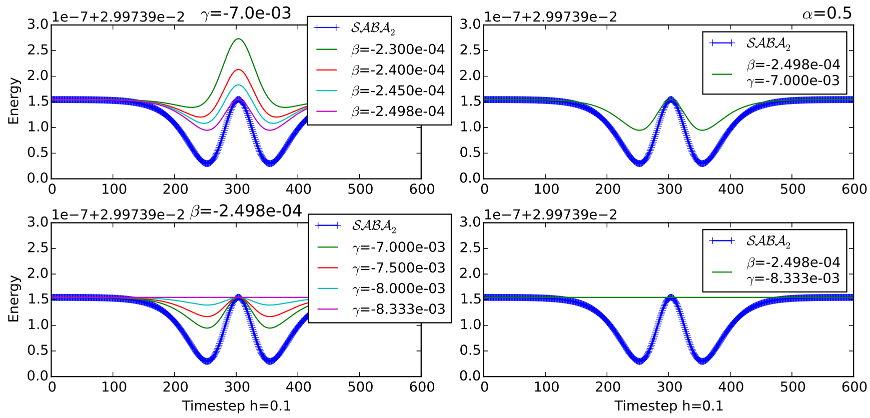

Figure 1. Some plots showing the manipulation of the numerical oscillations around a fixed energy value for the Hamiltonian pendulum. The fixed oscillating orbit corresponds to the numerical solution of the SABA2 integrator from Laskar and Robutel (2001). The oscillations of the flat line in the last plot have order 10^{-12}. All plots have initial condition (q0, p0) = (3.1, 0.0) and stepsize h = 0.1.

References

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